>>> Level-set frameworks

Name: Thibaut METIVET

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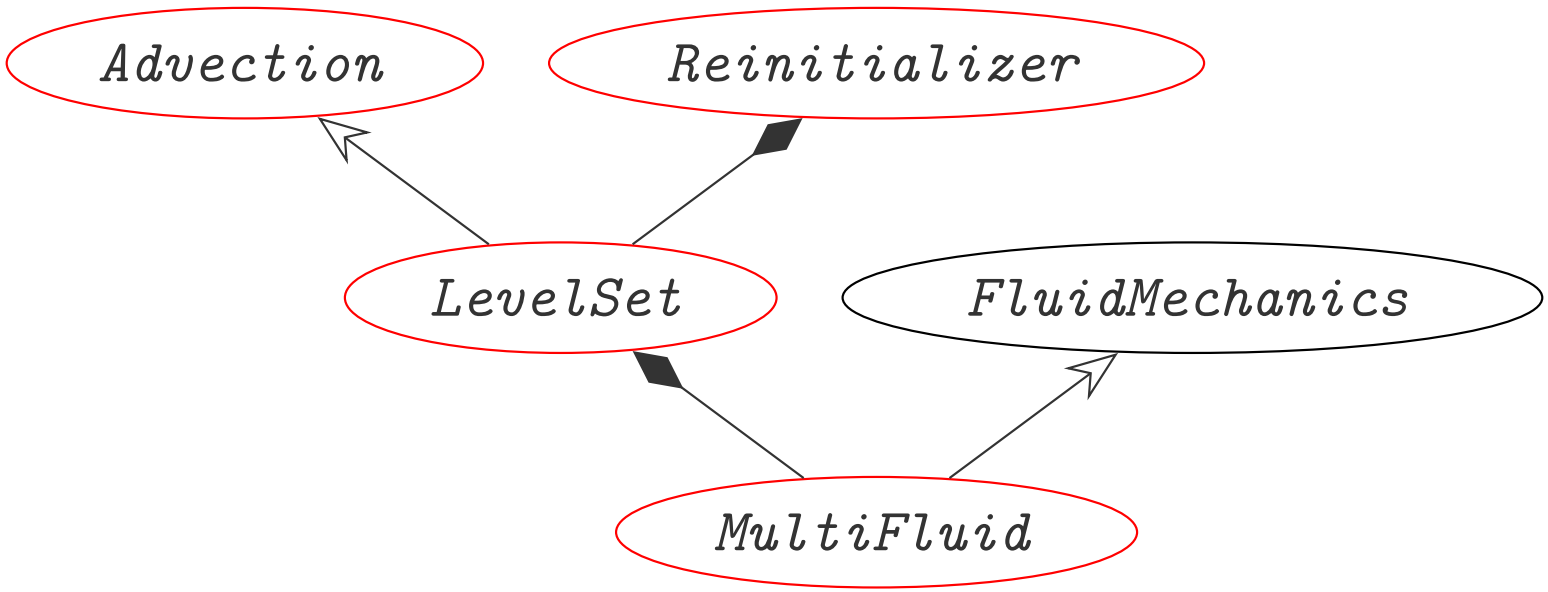
Advection framework

Levelset framework

Multifluid framework

>>> Frameworks


Advection
Reinitializer
LevelSet
FluidMechanics
MultiFluid

Advection framework

>>> Mathematical problem

$$\partial_t \phi + \mathbf{u} \cdot \nabla \phi - \nabla \cdot (D \, \nabla \phi) + R \, \phi = 0$$

$\mathbf{u}$: advection velocity
D: diffusion coefficient
R: reaction coefficient

Variational formulation

$\forall \, \psi \in H^1(\Omega)$,

$$\int_\Omega (\partial_t \phi + \mathbf{u} \cdot \nabla \phi) \, \psi + \int_\Omega D \, \nabla \phi \cdot \nabla \psi + \int_\Omega R \, \phi \, \psi = 0$$

Time

Backward differentiation formula scheme

$$\partial_t \phi \rightarrow \sum_i^n \frac{\alpha_i}{\delta t} \phi^{n+1-i}$$

Space

Lagrange P_n finite-elements

$\rightarrow$ need stabilization (GLS, SUPG, CIP)

Levelset framework

$\phi(\mathbf{x})$ **signed** distance function to interface: $|\boldsymbol{\nabla}\phi| = 1$

Evolution

Pure advection: $\partial_t \phi + \mathbf{u} \cdot \nabla \phi = 0$

Toolbox

feelpp_toolbox_levelset_[2d/3d]

Initial value

Only $\phi = 0$ iso-level relevant e.g.

* circle: $\phi = \sqrt{(x - x_c)^2 + (y - y_c)^2} - r$
* ellipse: $\phi = \sqrt{\left(\frac{x - x_c}{a}\right)^2 + \left(\frac{y - y_c}{b}\right)^2} - 1$ (redistantiate)

$\rightarrow$ set in JSON file

>>> JSON file

```
{
    "Name": "LS",
    "Model": "Advection",
    "Parameters":
    {
        "x0": "0.", "y0": "0.", "a0": "1.", "b0": "2."
    },
    "InitialConditions":
    {
        "[levelset_prefix]":
        {
            "Dirichlet":
            {
                "":
                {
                    "expr":"sqrt( (x-x0)*(x-x0)/(a0*a0) + (y-y0)*(y-y0)/(b0*b0) ) - 1
                            :x:x0:y:y0:a0:b0"
                }
            },
            "shapes":
            {
                "sphere1":
                {
                    "shape": "sphere",
                    "xc": "0",
                    "yc": "0",
                    "zc": "0",
                    "radius": "0.5"
                }
            }
        }
    }
}
```

>>> Geometrical quantities

Interface

$$\delta_\phi(\mathbf{x}) = \begin{cases} 0 & \text{if } \phi(\mathbf{x}) < -\varepsilon \\ \dfrac{1}{2\varepsilon}\left[1+\cos\left(\dfrac{\pi\phi}{\varepsilon}\right)\right] & \text{if } -\varepsilon \leq \phi(\mathbf{x}) \leq \varepsilon \\ 0 & \text{if } \phi(\mathbf{x}) > \varepsilon \end{cases}$$

```
element_levelset_ptrtype const& dirac() const;
```

Inner and outer

$$H_\phi(\mathbf{x}) = \begin{cases} 0 & \text{if } \phi(\mathbf{x}) < -\varepsilon \\ \dfrac{1}{2}\left[1+\dfrac{\phi}{\varepsilon}+\dfrac{1}{\pi}\sin\left(\dfrac{\pi\phi}{\varepsilon}\right)\right] & \text{if } -\varepsilon \leq \phi(\mathbf{x}) \leq \varepsilon \\ 1 & \text{if } \phi(\mathbf{x}) > \varepsilon \end{cases}$$

```
element_levelset_ptrtype const& heaviside() const;
```

>>> Geometrical quantities: related options

$$\int_\Gamma f(\mathbf{x}) \rightarrow \int_\Omega f(\mathbf{x})\, \delta_\phi(\mathbf{x})\, |\boldsymbol{\nabla}\phi| \;\text{ or }\; \int_\Omega f(\mathbf{x})\, \delta_{\frac{\phi}{|\boldsymbol{\nabla}\phi|}}(\mathbf{x})$$

$$\int_{\{\phi(\mathbf{x})\geq 0\}} f(\mathbf{x}) \rightarrow \int_\Omega f(\mathbf{x}) H_\phi(\mathbf{x})$$

Options

--[prefix].thickness-interface double: value of ε

--[prefix].use-regularized-phi bool: use $\frac{\phi}{|\boldsymbol{\nabla}\phi|}$

--[prefix].h-d-nodal-proj bool: use nodal projection (or L_2 projection)

>>> Normal and curvature

Normal

$$\mathbf{N} = \frac{\nabla \phi}{|\boldsymbol{\nabla} \phi|}$$

```
element_levelset_vectorial_ptrtype const& normal() const;
```

```
--[prefix].smooth-gradient bool: smooth ∇φ
```

Curvature

$$\kappa = \nabla \cdot \mathbf{N}$$

```
element_levelset_ptrtype const& curvature() const;
```

```
--[prefix].smooth-curvature bool: smooth κ
```

>>> Computing derivatives: projector

L_2 projector

$\Pi_{L_2}[f] = \hat{f} \in \mathbb{V} \subset L_2 \cap \mathcal{C}^0$ s.t.

$$\int_\Omega \hat{f}\, v = \int_\Omega f\, v, \quad \forall v \in \mathbb{V}$$

e.g. $\mathbf{N} = \Pi_{L_2}\left[\frac{\nabla\phi}{|\boldsymbol{\nabla}\phi|}\right]$

Need to have

$$(\text{polynomial order of } \mathbb{V}) \geq (\text{derivative order of } f)$$

Options

Standard backend options with prefixes

--[levelset_prefix].projector-l2.[backend_option]
--[levelset_prefix].projector-l2-vec.[backend_option]

>>> Computing derivatives: smoother

Smoother

For low-order discretization spaces, high-order derivatives become **inaccurate**

$\Rightarrow$ need to smooth out numerical errors

$\Pi_{sm}[f] = \tilde{f} \in \mathbb{W} \subset H^1 \cap \mathcal{C}^0$ s.t.

$$\int_\Omega \tilde{f}\, w + \int_\Omega \eta\, \nabla \tilde{f} \cdot \nabla w - \int_{\partial\Omega} \eta\, \nabla \tilde{f} \cdot \mathbf{n}\, w = \int_\Omega f\, w, \quad \forall w \in \mathbb{W}$$

$\rightarrow$ adds diffusion errors: need to tune η

Options

--[levelset_prefix].smoother.[backend_option]

--[levelset_prefix].smoother.smooth-coeff double: tune η

Same for smoother-vec.

>>> Redistanciation

After some iterations, need to restore $|\nabla \phi| = 1$

$\Rightarrow$ **fast-marching** or **Hamilton-Jacobi** method

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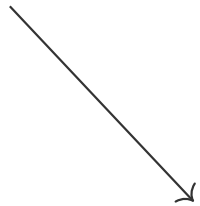

+ Fast $\mathcal{O}(N\log(N))$
+ "Exact"
- Limited to P_1 space
- Not fully parallel

Solve $\partial_\tau\phi = sign(\phi)\,(1 - |\nabla\phi|)$

+ Any P_N
+ Scales in parallel
- Can need many iterations
- No stop criterion
- Interface not anchored

>>> Redistanciation options

--[prefix].reinit-method **fm/hj**: select the redistanciation method

Fast-marching

--[prefix].fm-init-first-elts-strategy **int**: choose strategy to initialize the first values

0: do nothing

1: use $\frac{\phi}{|\nabla\phi|}$ on interface elements

2: use a few steps of Hamilton-Jacobi

Hamilton-Jacobi

--[prefix].reinit-hj.tol **double**: tolerance on relative rate of change for convergence

--[prefix].reinit-hj.max-iter **int**: max number of HJ iterations

>>> Physics enhancements

Stretch

If $\nabla \cdot \mathbf{u} = 0$, $s = |\boldsymbol{\nabla}\phi|$ accounts for the changes in surface measure

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But inaccurate + redistanciation $\Rightarrow s = 1$

$\rightarrow$ advect as an **independent** field

$$\partial_t s + \mathbf{u} \cdot \nabla s + (\mathbf{N} \otimes \mathbf{N}) : D(\mathbf{u})\, s = 0$$

--[prefix].use-stretch-augmented bool

--[prefix].reinit-stretch-augmented bool: reinitialize s to 1 but without changing its value near the interface

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Membrane mechanics

--[prefix].use-cauchy-augmented bool

Multifluid framework

Purpose

* Physical parameters and forces
* Couple level-set and fluid
  - Solve Navier-Stokes equation $\rightarrow \mathbf{u}^{(n+1)}, p^{(n+1)}$
  - Advect level set with $\mathbf{u}^{(n+1)} \rightarrow \phi^{(n+1)}$
  - Update fluid parameters and forces $\rightarrow \rho_\phi^{(n+1)}, \mu_\phi^{(n+1)}, \boldsymbol{F}_\phi^{(n+1)}$
* Manage redistantiation

Toolbox

feelpp_toolbox_multifluid_[2d/3d]

>>> Problem setup and physical parameters

Generic options

--multifluid.nfluids int: number of fluids considered (≥ 2)
--multifluid.[mesh options]

--multifluid.fluid.[fluid option]: fluid solver options
--multifluid.levelset[i].[levelset option]: levelset options

Physical parameters

--multifluid.fluid.[rho/mu]: outer fluid parameters
--multifluid.levelset[i].[rho/mu]: levelset i inner fluid parameters

>>> Forces

Based on

```
template<class LevelSetType>
class InterfaceForcesModel
{
    [...]
    virtual void build( std::string const& prefix,
                        levelset_ptrtype const& ls );
    void updateInterfaceForces( element_ptrtype & F,
                                bool overwrite = false) const;
    [...]
private:
    virtual void updateInterfaceForcesImpl( element_ptrtype & F )
        const =0;
    [...]
};
```

and registered in MultiFluid force factory.

Available forces

--multifluid.levelset[i].interface-forces-model string: enable given force model

* helfrich
* inextensibility-force
* linear-elastic-force
* skalak-force

A simple demo...